

Exam introduction to Energy and Environment 1
25-1-2017, 14-17 in NB 5118.-156 (Basement Nijenborgh 4)

Write (clearly!) your name, birth date, and study number on the first sheet of paper; on every following paper your name.

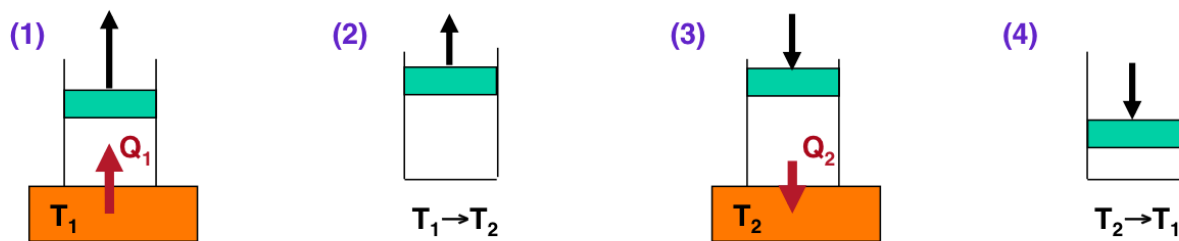
Use of any supporting material is not allowed, except for a simple calculator. Ask for extra paper if needed.

General recommendations:

- Calculate, where applicable, symbolically as long as possible; only fill in numbers in the final step(s)!
- Make that every calculation or line of reasoning is comprehensible, also for the corrector
- Everywhere where explanations are requested, be as complete and concrete as possible, but "complete" is not the same as "lengthy"

Question 1: The Carnot Cycle
(indication: 8 of a total of 30 points)

The Carnot cycle, with a perfect gas and an ideal, frictionless machine:



1 Isothermal expansion with supply of heat Q_1 from the heat bath at T_1

2 Adiabatic expansion without heat supply \Rightarrow cooling to T_2

3 Isothermal compression with removal of heat Q_2 to heat bath at T_2

4 Adiabatic compression without heat removal \Rightarrow heating to T_1

(a) Draw a clear schematic PV-diagram for this reversible cycle, for which the end of the movement at (4) brings the situation exactly back to the start of situation (1). Indicate clearly the work performed and the heat flows (with the same names as in the drawings above), with their directions (heat to or from the system, work by or on the system).

(b) For each branch of the diagram, calculate the exchanged heat and work, expressed in volumes and/or temperatures (or if either of these is zero, state that).

(c) Define the efficiency of this engine, and derive the well-known expression for the Carnot efficiency.

You may use: for a perfect gas $PV=RT$, for an adiabat $PV^\gamma = \text{constant}$ (with γ the ratio of the specific heat at constant pressure and at constant volume).

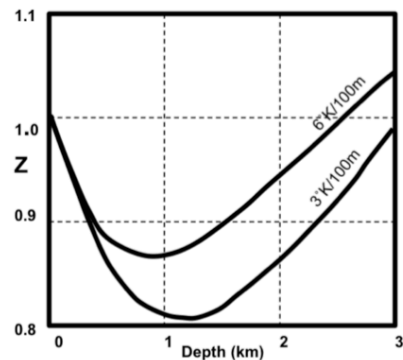
Question 2: Geo-energy:
(indication: 5 of a total of 30 points)

The natural gas field "Zernike" produces 3 million m^3 gas per day at standard conditions (1 Bar and 15°C) and for simplicity we assume it is pure methane. Methane has a combustion heat of $35 \text{ MJ}/\text{m}^3$.

(a) What is the power of this gas field in MW?

The field is at a depth of 2 km, in a hydrostatic pressure regime, and is situated in an area with a geothermal gradient of $3^\circ\text{C}/100\text{m}$. The standard conditions at the surface are a temperature of $T_s=15^\circ\text{C}$ and atmospheric pressure p_s of 1 bar. The diagram shows the value of the

Compressibility Factor for Methane



compressibility factor Z as a function of depth for methane. The gas expansion factor is given by:

$$E = (T_s/p_s) \cdot (p/ZT)$$

(b) Calculate the volume that has been extracted from the field under reservoir conditions in five years of production.

The field consists of pure sandstone with an average porosity of 20% and a gas saturation of 90%. The area of the field is 10 km² and its thickness is 80 meters.

(c) For how long can the given rate of production be maintained (assuming that all gas eventually can be produced)?

Question 3. A double glazed window
(indication: 8 of a total of 30 points)

Fourier's law (in one dimension) for heat conduction through a material is: $q = -kA \frac{\Delta T}{d}$

with ΔT the temperature difference [K or °C] between the two sides of the material, d the thickness of the material [m] and k the thermal conductivity [W/(Km)]. Finally, q is the resulting heat current [W].

When two materials "1" and "2", with different k 's and d 's, are connected, the heat flow through the two materials caused by a temperature gradient over the materials is the same in each of the two materials.

a) Proof, using that principle, that the relation between the temperature difference over this combination of two materials and the heat flow through the materials can be expressed as:

$$\Delta T = q \left(\frac{d_1}{k_1 A} + \frac{d_2}{k_2 A} \right) \equiv q (R_1 + R_2) \quad \text{with } R_1 \text{ and } R_2 \text{ the heat resistances of the two materials}$$

The concept of heat resistance is very useful when computing the heat flow through combinations of two or more layers. An example is a 'double glass' window, which consists of three layers: 4 mm glass, 6 mm air and again 4 mm glass. The inside temperature is 20 °C, the outside temperature is 0 °C. The thermal conductivity of glass is 1.4 W/(Km) and of air 0.026 W/(Km). We neglect radiation and convection effects.

b) Calculate the total heat resistance of this double glass window for 1 m²

c) Sketch the temperature gradient through the three materials (indicate clearly inside and outside).

d) Calculate the heat loss [W] for the double glass window with an area of $A = 3 \text{ m}^2$.

e) Calculate how thick a single glass window would have to be to have the same heat loss.

Question 4: Nuclear fission
(indication: 4 of a total of 30 points)

(a) Calculate/estimate the energy released by splitting a ²³⁵Uranium nucleus, by using the figure below, and explain your estimate.

(b) Use the result of (a) (if you did not find an answer, use 185 MeV, which is by the way not the correct answer). How long does it take to use up 1 kg of ²³⁵Uranium in a thermal reactor capacity of 100 MW?

(c) For nuclear mass numbers below 10, the binding energy per nucleon goes steeply down (not shown in the graph). What would be the binding energy for Hydrogen?

Data: Avogadro's number: 6.10^{23} ; electron charge: $1.6.10^{-19} \text{ C}$

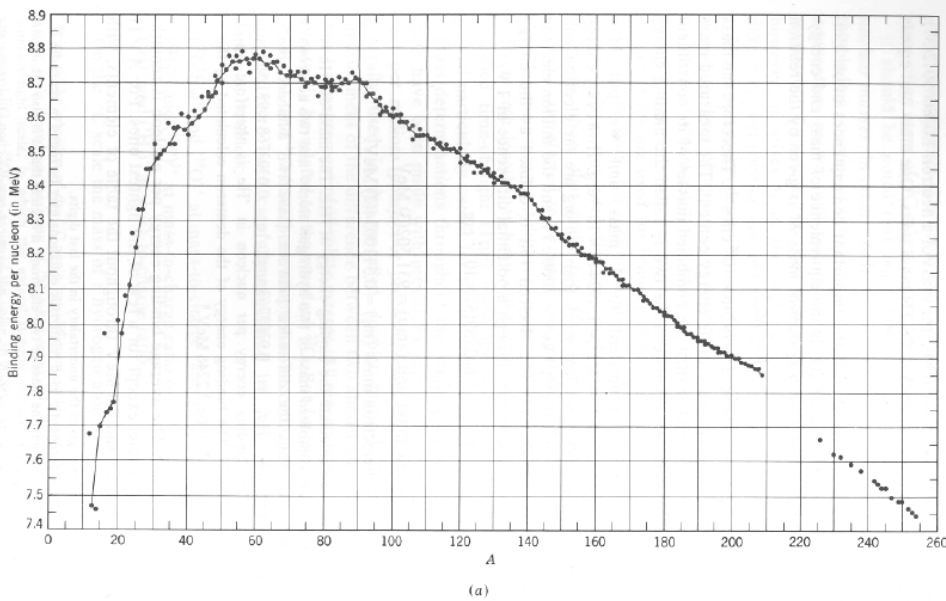
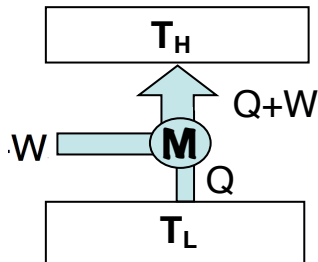


Figure: Binding energy per nucleon (in MeV) as a function of the nuclear mass (A)

Question 5 Efficiency

(indication: 5 of a total of 30 points)

Consider the schematic drawing of a certain system below: two heat baths at higher (T_H) and lower (T_L) temperature, respectively, and some engine that is pumping heat Q from the low temperature bath, and heat $Q+W$ to the high temperature bath, using work W . The system is supposed to be fully reversible:



a) Give the entropy changes of both heat baths.

b) This schematic drawing may represent either a heat pump, or a refrigerator. Explain the different purposes of these two systems, and give the expression for the efficiencies for both systems (expressed in Q and W).

c) What is the total change of entropy for a reversible system according to the second law of thermodynamics?

d) Use (a) – (c) to find the efficiency for the system in use as refrigerator: $\eta = \frac{1}{\frac{T_H}{T_C} - 1}$

This now being a refrigerator, we want to maintain the inside temperature at 4°C , and therefore have to "pump" every second the heat that leaks in Q to the warm grid with temperature T_H on the outside (the back) of the refrigerator. The amount of "leak heat" depends on the outside temperature T_B as $Q = 20(T_B - T_C)$ Joules. The temperature T_H of the refrigerator grid is obviously higher than T_B , we assume this difference to be 10°C .

e) Calculate the (electric) power of the refrigerator, that is the work W per second, for a room with $T_B = 20^\circ\text{C}$. How much kWh is that per year?